Grade 6 - Unit D - Expressions, Equations & Rational Numbers

Unit Focus

Students begin the unit by working with linear equations that have single occurrences of one variable. They represent relationships with tape diagrams and with linear equations, explaining correspondences between these representations. They examine values that make a given linear equation true or false, and what it means for a number to be a solution to an equation. Balanced and unbalanced "hanger diagrams" are introduced as a way to reason about solving the linear equations of the first section. Students then write expressions with whole-number exponents and whole-number, fraction, or variable bases. They evaluate such expressions, using properties of exponents strategically. They understand that a solution to an equation in one variable is a number that makes the equation true when the number is substituted for all instances of the variable. They represent algebraic expressions and equations in order to solve problems.

In the second part of the unit, signed numbers are introduced. Students begin by considering examples of positive and negative temperatures, plotting each temperature on a vertical number line on which 0 is the only label. Next, they consider examples of positive and negative numbers used to denote height relative to sea level. In the second lesson, they plot positive and negative numbers on horizontal number lines, including "opposites"—pairs of numbers that are the same distance from zero. They use "less than," "greater than," and the corresponding symbols to describe the relationship of two signed numbers. They learn that the absolute value of a number is its distance from zero, how to use absolute value notation, and that opposites have the same absolute value because they have the same distance from zero. In comparing two signed numbers, students distinguish between magnitude (the absolute value of a number) and order (relative position on the number line), distinguishing between "greater than" and "greater absolute value," and "less than" and "smaller absolute value." Students examine opposites of numbers, noticing that the opposite of a negative number is positive.

Students graph simple inequalities in one variable on the number line, using a circle or disk to indicate when a given point is, respectively, excluded or included. Students represent situations that involve inequalities, symbolically and with the number line, understanding that there may be infinitely many solutions for an inequality. They interpret and graph solutions in contexts (MP2), understanding that some results do not make sense in some contexts, and thus the graph of a solution might be different from the graph of the related symbolic inequality.

In this unit, students work in all four quadrants of the coordinate plane, plotting pairs of signed number coordinates in the plane. They understand that for a given data set, there are more and less strategic choices for the scale and extent of a set of axes. They understand the correspondence between the signs of a pair of coordinates and the quadrant of the corresponding point. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal and vertical distances between two points.

Stage 1: Desired Results - Key Understandings

Standard(s)	Transfer
Standards • Common Core ○ *Mathematics: 6* ▪ Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. *(CCSS.MATH.CONTENT.6.RP.A.3)*	*Students will be able to independently use their learning to...* **T1** Construct viable arguments using clear and appropriate mathematical language and critique the reasoning of others. **T2** Identify and generalize patterns and structure in numbers, expressions, data and objects.

Stage 1: Desired Results - Key Understandings

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. *(CCSS.MATH.CONTENT.6.RP.A.3A)*
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? *(CCSS.MATH.CONTENT.6.RP.A.3B)*
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. *(CCSS.MATH.CONTENT.6.RP.A.3C)*
- Apply and extend previous understandings of numbers to the system of rational numbers.
- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. *(CCSS.MATH.CONTENT.6.NS.C.5)*
- Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. *(CCSS.MATH.CONTENT.6.NS.C.6)*
- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. *(CCSS.MATH.CONTENT.6.NS.C.6A)*
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. *(CCSS.MATH.CONTENT.6.NS.C.6B)*
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. *(CCSS.MATH.CONTENT.6.NS.C.6C)*
- Understand ordering and absolute value of rational numbers. *(CCSS.MATH.CONTENT.6.NS.C.7)*
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right. *(CCSS.MATH.CONTENT.6.NS.C.7A)*
- Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3 oC > -7 oC to express the fact that -3 oC is warmer than -7 oC. *(CCSS.MATH.CONTENT.6.NS.C.7B)*
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars. *(CCSS.MATH.CONTENT.6.NS.C.7C)*

Meaning

Understanding(s)	Essential Question(s)
Students will understand that... **U1** Mathematicians construct viable arguments to explain problems, solutions, and mathematical representations. **U2** Mathematicians see patterns to make generalizations about structures and relationships.	*Students will keep considering...* **Q1** How can I strengthen my argument and reasoning? **Q2** What generalizations can be made from this pattern?

Acquisition of Knowledge and Skill

Knowledge	Skill(s)
Students will know... **K1** the difference between an equation, inequality and an expression. **K2** variables represent unknown numbers in expressions and equations. **K3** inequalities have many solutions **K4** the solution(s) of an equation are the value(s) of the variable(s) that make the equation true **K5** which is the dependent and independent variable in an equation **K6** the difference between a positive number, a negative number, and zero **K7** the absolute value of a number is its distance from zero **K8** exponents represent the repeated multiplication of a value **K9** each of the four quadrants of a coordinate grid are represented by different positive and/or negative x- and y- coordinates **K10** Vocabulary: operation, variable, coefficient, value, equation, equivalent expressions, term, distributive property,	*Students will be skilled at...* **S1** using variables to write expressions for real-world and mathematical situations **S2** creating and solving equations that are based on real world and mathematical situations **S3** Writing an inequality in the form x>c or x<c **S4** using substitution to solve equations or inequalities. **S5** analyzing the relationship between dependent and independent variables **S6** using multiple strategies to solve equations/inequalities. **S7** placing integers on a number line **S8** comparing and ordering integers **S9** using equations to solve problems involving percentages

Stage 1: Desired Results - Key Understandings

▪ Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars. *(CCSS.MATH.CONTENT.6.NS.C.7D)* ▪ Apply and extend previous understandings of numbers to the system of rational numbers. ▪ Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. *(CCSS.MATH.CONTENT.6.NS.C.8)* ▪ Apply and extend previous understandings of arithmetic to algebraic expressions. ▪ Write and evaluate numerical expressions involving whole-number exponents. *(CCSS.MATH.CONTENT.6.EE.A.1)* ▪ Write, read, and evaluate expressions in which letters stand for numbers. *(CCSS.MATH.CONTENT.6.EE.A.2)* ▪ Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation Subtract y from 5 as 5 - y. *(CCSS.MATH.CONTENT.6.EE.A.2A)* ▪ Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. *(CCSS.MATH.CONTENT.6.EE.A.2B)* ▪ Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2. *(CCSS.MATH.CONTENT.6.EE.A.2C)* ▪ Apply and extend previous understandings of arithmetic to algebraic expressions. ▪ Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. *(CCSS.MATH.CONTENT.6.EE.A.3)* ▪ Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. *(CCSS.MATH.CONTENT.6.EE.A.4)* ▪ Reason about and solve one-variable equations and inequalities. ▪ Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to	exponent, to the power, base (of an exponent), independent variable, dependent variable, coordinate plane, horizontal axis, vertical axis, plot, positive number, negative number, below zero elevation, sea level, opposite numbers, rational numbers, sign, inequality, greater than, less than, absolute value, maximum, minimum, solution to an inequality, quadrant, x-coordinate, y-coordinate **K11** there is an order to how operations are used when solving an expression or equation	**S10** generating equivalent numerical expressions that are related by the distributive property **S11** understanding expressions using exponents **S12** plotting and identifying points in all 4 quadrants of a coordinate grid **S13** determining the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate. **S14** solving expressions and equations following the order of operations. **S15** solving problems using the absolute value of a number

Stage 1: Desired Results - Key Understandings

determine whether a given number in a specified set makes an equation or inequality true. *(CCSS.MATH.CONTENT.6.EE.B.5)* ▪ Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. *(CCSS.MATH.CONTENT.6.EE.B.6)* ▪ Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. *(CCSS.MATH.CONTENT.6.EE.B.7)* ▪ Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. *(CCSS.MATH.CONTENT.6.EE.B.8)* ▪ Represent and analyze quantitative relationships between dependent and independent variables. ▪ Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time. *(CCSS.MATH.CONTENT.6.EE.C.9)* ▪ Solve real-world and mathematical problems involving area, surface area, and volume. ▪ Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. *(CCSS.MATH.CONTENT.6.G.A.3)* ▪ Mathematical Practices ▪ Construct viable arguments and critique the reasoning of others. *(CCSS.MATH.MP.3)* ▪ Look for and make use of structure. *(CCSS.MATH.MP.7)* **Madison Public Schools Profile of a Graduate** Analyzing: Examining information/data/evidence from multiple sources to identify possible underlying assumptions, patterns, and relationships in order to make inferences. *(POG.1.2)*		