Unit 3 - *Quadratic Equations and Complex Numbers*

Overview

The goal of this unit is for students to become fluent in interpreting, solving, and graphing quadratic functions with rational and irrational solutions as well as complex roots. The connection between completing the square and equations of circles is made. Students model using quadratic functions.

21st Century Capacities: Analyzing

Stage 1 - Desired Results

ESTABLISHED GOALS/ STANDARDS

MP 1 Make sense sense of problems and persevere in solving them
MP4 Model with Mathematics
MP6 Attend to precision
MP7 Look for and make use of structure

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
N.CN.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3(+) Find the conjugate of a complex number; use conjugates of a complex number to find quotients of complex numbers.
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
F.BF.3 See unit 1
A.SSE.3 Choose and produce an equivalent form of an expression

Transfer:

Students will be able to independently use their learning in new situations to...

1. Model relationships among quantities.
2. Manipulate equations and expressions to create order and establish relationships. (Analyzing)
3. Draw conclusions about graphs, shapes, equations, or objects. (Analyzing)

Meaning:

UNDERSTANDINGS: *Students will understand that:*

1. Mathematicians examine relationships to discern a pattern, generalizations, or structure.
2. Mathematicians can describe patterns, relations, and/or functions to access strategies to solve problems.
3. Mathematicians use models to represent and make meaning of quantitative relationships.
4. Mathematicians analyze change and make predictions in various contexts.

ESSENTIAL QUESTIONS: *Students will explore and address these recurring questions:*

A. What is the most efficient way to solve this problem?
B. What can we learn about the relationship from the function and its graph?
C. How do quadratic equations model real world phenomena?
D. How do you express and describe a pattern and use it to make predictions and solve a problem?

to reveal and explain properties of the quantity represented by the expression* A.SSE.3a Factor a quadratic expression to reveal zeros of the function it defines. A.SSE.3b Complete the square in a quadratic expression to reveal the maximum value of the function it defines. N.RN.3 Explain why the sum or product of two rational numbers is rational and the sum of a rational and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous roots may arise. A.REI.4 Solve quadratic equations in one variable. A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.7b Graph square root A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	***Acquisition:*** *Students will know…* 1. The Fundamental Theorem of Algebra 2. i = √ -1 and the cyclical nature of i 3. a + bi form of a complex number 4. The general and standard form of a circle 5. Vocabulary: complex number, imaginary unit, conjugates, zeros, discriminant	*Students will be skilled at…* 1. Combining and simplifying radical expressions including those with rational fractions and cube roots 2. Solving quadratic equations by factoring, completing the square, graphing and the quadratic formula 3. Writing quadratic equations in standard form or vertex form 4. Finding the axis of symmetry, domain, range, min or max, intercepts of a quadratic function 5. Using the discriminant to find the number and type of solutions 6. Identifying real and imaginary components of complex numbers 7. Adding, subtracting, multiplying, and dividing complex numbers 8. Graphing a circle given the equation or vice versa 9. Determining tangent lines to a circle 10. Using transformations on the parent quadratic function to graph 11. Graphing and solving nonlinear systems of equations and inequalities 12. Solving, graphing, and transforming radical functions