Unit C - *Vectors*

Overview

This two part unit explores applications of trignometry, most importantly vectors.

In part 1 the Laws of Sines and Cosines are derived, allowing us to solve for sides and angles in oblique (non-right) triangles. Students need to be aware that in the SSA case, there could be no, one, or two possible triangles and why this happens. Area of triangles is also covered at this time for the SAS and SSS cases (Heron's Formula). After oblique triangles, most of the unit is spent on vectors, quantities with both magnitude and direction (velocity, force, etc). Students will learn how to express them in component form as well as a magnitude and direction angle. They will learn how to perform several operations on vectors, including the dot product. This operation is used to find angles between vectors and projections of vectors. Many applications of vectors are discussed, including plane and wind problems, force balancing, weights on ramps, and work. The unit finishes with new topic, complex numbers. By converting from a + bi form to a trigonometric form, some calculations (raising to powers and finding roots) can be done much quicker by using DeMoivre's Theorem.

Part 2 extends the topic of vectors to 3-dimensional space. The unit begins by discussing the 3D coordinate system, including how to plot points, find distance between points, midpoints and equations of spheres. Vectors are useful in 3D to determine if points are collinear. Angles between vectors is revisited, along with the dot product. A new operation is taught, the cross product, which is only possible in 3D space. Students will learn that this operation creates a new vectors which is normal (perpendicular) to the plane containing the two original vectors and can be used to find area and volume of parallelogram-type figures. Finally, vectors are used to determine equations of lines and planes in 3D space and determining the distance between a point and a plane.

21st Century Capacities: Synthesizing

Stage 1 - Desired Results

ESTABLISHED GOALS/ STANDARDS	***Transfer:***
MP 1 Make sense sense of problems and persevere in solving them **MP2** Reason abstractly and quantitatively **MP4** Model with Mathematics	*Students will be able to independently use their learning in new situations to...* 1. Make sense of a problem, initiate a plan, execute it, and evaluate the reasonableness of the solution. (synthesizing) 2. Use appropriate tools to make reaching solutions more efficient, accessible and accurate.

CCSS.MATH.CONTENT.HSN.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

CCSS.MATH.CONTENT.HSN.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.*

CCSS.MATH.CONTENT.HSN.VM.A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $\|\boldsymbol{v}\|$, v).

CCSS.MATH.CONTENT.HSN.VM.A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

CCSS.MATH.CONTENT.HSN.VM.A.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

CCSS.MATH.CONTENT.HSN.VM.B.4 (+) Add and subtract vectors.

CCSS.MATH.CONTENT.HSN.VM.B.4.A Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

3. Apply familiar mathematical concepts to a new problem or apply a new concept to rework a familiar problem.

Meaning:

UNDERSTANDINGS: *Students will understand that:*

1. Mathematicians flexibly use different tools, strategies, and operations to build conceptual knowledge or solve problems.
2. Mathematicians apply the mathematics they know to solve problems occurring in everyday life.
3. Mathematicians use models to represent and make meaning of quantitative relationships.

ESSENTIAL QUESTIONS: *Students will explore & address these recurring questions:*

A. How can I use what I know to help me find what is missing?
B. What math tools/models/strategies can I use to solve the problem?
C. What have I seen in the past that might help me now?

Acquisition:

Students will know…

1. Trigonometry can be used to solve for sides and angles in non-right triangles
2. Vectors are quantities with both direction and magnitude, while scalars have only magnitude
3. Vectors can be used to solve a variety of real world problems.
4. Complex numbers can be expressed using trigonometry
5. How the 3D coordinate system is defined
6. The cross product is only defined in 3D space and is important for determining equations of planes
7. Vocabulary: Law of Sines, Law of cosines, oblique triangles, Heron's

Students will be skilled at…

1. Applying the Law of Sines and Cosines and finding areas of triangles.
2. Converting vectors from component form to direction and magnitude using trigonometry
3. Performing operations on vectors, including dot product and projections
4. Finding angles between vectors and determining orthogonality
5. Using DeMoivre's theorem to raise complex numbers to powers and find roots of complex numbers
6. Finding distances, midpoints, and equations of spheres in 3D
7. Determining if 3D vectors are parallel

CCSS.MATH.CONTENT.HSN.VM.B.4.B Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. CCSS.MATH.CONTENT.HSN.VM.B.4.C Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. CCSS.MATH.CONTENT.HSN.VM.B.5 (+) Multiply a vector by a scalar. CCSS.MATH.CONTENT.HSN.VM.B.5.A Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. CCSS.MATH.CONTENT.HSN.VM.B.5.B Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	formula, vector, scalar, components, unit vector, standard unit vector, dot product, orthogonal, projection, work, DeMoivre's Theroem, collinear, cross product, parallelepiped, triple scalar product, parametric and symmetric equations, normal	and if points are collinear in 3D space 8. Determining and using the cross product and triple scalar product to solve problems 9. Finding parametric and symmetric equations of lines in 3D 10. Determining equations of planes in 3D 11. Finding the distance between a point and a plane