

## Unit E - Sequences, Series, and Probability

### Overview

This unit is all about recognizing patterns. Formulas for  $n$ th terms and summations of arithmetic and geometric sequences are derived. The concept of induction is introduced as a way to prove other formulas for series, divisibility, and inequalities. Then the binomial theorem is taught as an application of another pattern. Finally, combinations and permutations are covered and these counting principles are used to find probabilities.

**21<sup>st</sup> Century Capacities:** Analyzing

### Stage 1 - Desired Results

<p><b>ESTABLISHED GOALS/ STANDARDS</b></p> <p><b>MP 1</b> Make sense of problems and persevere in solving them  <b>MP2</b> Reason abstractly and quantitatively  <b>MP7</b> Look for and make use of structure</p> <p>CCSS.MATH.CONTENT.HSA.SSE.B.4                  Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example, calculate mortgage payments.*</i></p> <p>CCSS.MATH.CONTENT.HSA.APR.C.5                  (+) Know and apply the Binomial Theorem for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>1</sup></p> <p>CCSS.MATH.CONTENT.HSF.IF.A.3</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th colspan="2" style="background-color: #D3D3D3; text-align: center; padding: 5px;"><b>Transfer:</b></th> </tr> <tr> <td colspan="2" style="padding: 5px;"><i>Students will be able to independently use their learning in new situations to...</i></td> </tr> <tr> <td colspan="2" style="padding: 5px;"> <ol style="list-style-type: none"> <li>1. Manipulate equations/expressions or objects to create order and establish relationships.</li> <li>2. Represent and interpret patterns in numbers, data and objects. (Analyzing)</li> <li>3. Use appropriate tools to make reaching solutions more efficient, accessible and accurate.</li> </ol> </td> </tr> <tr> <th colspan="2" style="background-color: #D3D3D3; text-align: center; padding: 5px;"><b>Meaning:</b></th> </tr> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> <p><b>UNDERSTANDINGS:</b> <i>Students will understand that:</i></p> <ol style="list-style-type: none"> <li>1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions.</li> <li>2. Mathematicians examine relationships to discern a pattern, generalizations, or structure.</li> </ol> </td> <td style="width: 50%; padding: 5px; vertical-align: top;"> <p><b>ESSENTIAL QUESTIONS:</b> <i>Students will explore &amp; address these recurring questions:</i></p> <ol style="list-style-type: none"> <li>A. How do you express and describe a pattern and use it to make predictions and solve a problem?</li> <li>B. How can understanding a pattern help me?</li> <li>C. What is the most efficient way to solve this problem?</li> </ol> </td> </tr> <tr> <th colspan="2" style="background-color: #D3D3D3; text-align: center; padding: 5px;"><b>Acquisition:</b></th> </tr> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> <p><i>Students will know...</i></p> <ol style="list-style-type: none"> <li>1. How to express a sequence explicitly and recursively</li> </ol> </td> <td style="width: 50%; padding: 5px; vertical-align: top;"> <p><i>Students will be skilled at...</i></p> <ol style="list-style-type: none"> <li>1. Using <math>n</math>th term formulas to find specific terms of a sequence</li> </ol> </td> </tr> </table>	<b>Transfer:</b>		<i>Students will be able to independently use their learning in new situations to...</i>		<ol style="list-style-type: none"> <li>1. Manipulate equations/expressions or objects to create order and establish relationships.</li> <li>2. Represent and interpret patterns in numbers, data and objects. (Analyzing)</li> <li>3. Use appropriate tools to make reaching solutions more efficient, accessible and accurate.</li> </ol>		<b>Meaning:</b>		<p><b>UNDERSTANDINGS:</b> <i>Students will understand that:</i></p> <ol style="list-style-type: none"> <li>1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions.</li> <li>2. Mathematicians examine relationships to discern a pattern, generalizations, or structure.</li> </ol>	<p><b>ESSENTIAL QUESTIONS:</b> <i>Students will explore &amp; address these recurring questions:</i></p> <ol style="list-style-type: none"> <li>A. How do you express and describe a pattern and use it to make predictions and solve a problem?</li> <li>B. How can understanding a pattern help me?</li> <li>C. What is the most efficient way to solve this problem?</li> </ol>	<b>Acquisition:</b>		<p><i>Students will know...</i></p> <ol style="list-style-type: none"> <li>1. How to express a sequence explicitly and recursively</li> </ol>	<p><i>Students will be skilled at...</i></p> <ol style="list-style-type: none"> <li>1. Using <math>n</math>th term formulas to find specific terms of a sequence</li> </ol>
<b>Transfer:</b>															
<i>Students will be able to independently use their learning in new situations to...</i>															
<ol style="list-style-type: none"> <li>1. Manipulate equations/expressions or objects to create order and establish relationships.</li> <li>2. Represent and interpret patterns in numbers, data and objects. (Analyzing)</li> <li>3. Use appropriate tools to make reaching solutions more efficient, accessible and accurate.</li> </ol>															
<b>Meaning:</b>															
<p><b>UNDERSTANDINGS:</b> <i>Students will understand that:</i></p> <ol style="list-style-type: none"> <li>1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions.</li> <li>2. Mathematicians examine relationships to discern a pattern, generalizations, or structure.</li> </ol>	<p><b>ESSENTIAL QUESTIONS:</b> <i>Students will explore &amp; address these recurring questions:</i></p> <ol style="list-style-type: none"> <li>A. How do you express and describe a pattern and use it to make predictions and solve a problem?</li> <li>B. How can understanding a pattern help me?</li> <li>C. What is the most efficient way to solve this problem?</li> </ol>														
<b>Acquisition:</b>															
<p><i>Students will know...</i></p> <ol style="list-style-type: none"> <li>1. How to express a sequence explicitly and recursively</li> </ol>	<p><i>Students will be skilled at...</i></p> <ol style="list-style-type: none"> <li>1. Using <math>n</math>th term formulas to find specific terms of a sequence</li> </ol>														

## Pre-Calculus Level 1 Curriculum

<p>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p> <p>CCSS.MATH.CONTENT.HSF.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</p> <p>CCSS.MATH.CONTENT.HSS.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").</p> <p>CCSS.MATH.CONTENT.HSS.CP.A.2 Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>CCSS.MATH.CONTENT.HSS.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.</p>	<ol style="list-style-type: none"> <li>2. What summation and factorial notations mean</li> <li>3. How the process of induction works</li> <li>4. How to determine coefficients of a binomial expansion using counting principles or Pascal's triangle</li> <li>5. The various counting principles for events</li> <li>6. Vocabulary: Sequence, Series, Explicit, Recursive, Factorial, Summation, Arithmetic, Geometric, Induction, Binomial Expansion, Pascal's Triangle, Combinations, Permutations</li> </ol>	<ol style="list-style-type: none"> <li>2. Determined nth term formulas for arithmetic and geometric sequences</li> <li>3. Applying summation formulas for arithmetic and geometric sequences</li> <li>4. Applying induction to prove summation patterns and divisibility and inequalities</li> <li>5. Expanding a binomial using the binomial theorem</li> <li>6. Using counting principles to determine probabilities</li> </ol>
---	--	---