

## Unit B - Functions and their Graphs

### Overview

In this unit, students will be re-familiarized with the definition of functions, function notation, and operations on functions. Students will learn how to express the relationship between two variables as a function, how to graph these relationships on a graphing calculator and how to determine the optimal values of a function in an appropriate domain. Function families that will be covered in this unit include polynomials, rationals and radicals. A generalized transformation rule will be introduced and will be used for the remainder of the year.

**21<sup>st</sup> Century Capacities:** Synthesizing, Analyzing

### Stage 1 - Desired Results

**ESTABLISHED GOALS/ STANDARDS**

**MP2** Reason abstractly and quantitatively  
**MP4** Model with Mathematics  
**MP5** Use appropriate tools strategically

CCSS.MATH.CONTENT.HSA.SSE.B.3.B  
 Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  
 CCSS.MATH.CONTENT.HSA.CED.A.2  
 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

CCSS.MATH.CONTENT.HSA.CED.A.3  
 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  
 CCSS.MATH.CONTENT.HSA.REI.D.10  
 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  
 CCSS.MATH.CONTENT.HSA.REI.D.11

***Transfer:***

*Students will be able to independently use their learning in new situations to...*

1. Model relationships among quantities (analyzing).
2. Make sense of a problem, initiate a plan, execute it, and evaluate the reasonableness of the solution (synthesizing).
3. Use appropriate tools to make reaching solutions more efficient, accessible and accurate (synthesizing).

***Meaning:***

**UNDERSTANDINGS:** *Students will understand that:*

1. Mathematicians flexibly use different tools, strategies, and operations to build conceptual knowledge or solve problems.
2. Mathematicians apply the mathematics they know to solve problems occurring in

**ESSENTIAL QUESTIONS:** *Students will explore & address these recurring questions:*

- A. How can I break a problem down into manageable parts?
- B. What does the solution tell me?
- C. What is the most efficient way to solve this problem?
- D. What math tools/models/strategies can I use to solve the problem?

## Pre-Calculus Level 2 Curriculum

<p>Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*</p> <p>CCSS.MATH.CONTENT.HSF.IF.A.1</p> <p>Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>CCSS.MATH.CONTENT.HSF.IF.A.2</p>	<p>everyday life.</p> <ol style="list-style-type: none"> <li>3. Mathematicians create or use models to examine, describe, solve and/or make predictions.</li> <li>4. Mathematicians can describe patterns, relations, and/or functions to access strategies to solve problems.</li> <li>5. Mathematicians represent and analyze mathematical situations and structures using algebraic symbols to communicate thinking.</li> </ol>	
<p>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>CCSS.MATH.CONTENT.HSF.IF.B.4</p> <p>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</p> <p>CCSS.MATH.CONTENT.HSF.IF.B.5</p> <p>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</p> <p>CCSS.MATH.CONTENT.HSF.IF.B.6</p> <p>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</p> <p>CCSS.MATH.CONTENT.HSF.IF.C.7</p> <p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</p> <p>CCSS.MATH.CONTENT.HSF.IF.C.7.A</p> <p>Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>CCSS.MATH.CONTENT.HSF.IF.C.7.B</p>	<p style="text-align: center;"><b>Acquisition:</b></p> <p><i>Students will know...</i></p> <ol style="list-style-type: none"> <li>1. Library/family of functions</li> <li>2. graphing techniques</li> <li>3. Domain, range and intercepts of a function</li> <li>4. Fundamental Theorem of algebra</li> <li>5. Conjugate Roots Theorem</li> <li>6. Vocabulary: vertical line test, local/relative extrema, minima, maxima, domain, range, polynomial function, rational function, asymptotes</li> </ol>	<p><i>Students will be skilled at...</i></p> <ol style="list-style-type: none"> <li>1. Using function notation</li> <li>2. Determining if a relation is a function</li> <li>3. Describing the graph of a function</li> <li>4. Algebraically determine whether a function is even or odd</li> <li>5. Applying transformations to the graphs of parent functions</li> <li>6. Combining functions (add,sub,mult, div)</li> <li>7. Function composition</li> <li>8. Determining the interval on which a function is increasing and decreasing</li> <li>9. Writing function rules to model real-world scenarios especially optimization</li> <li>10. Determining the real domain for a real-world scenario</li> </ol>

## Pre-Calculus Level 2 Curriculum

<p>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. CCSS.MATH.CONTENT.HSF.IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. CCSS.MATH.CONTENT.HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. CCSS.MATH.CONTENT.HSF.IF.C.8.A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. CCSS.MATH.CONTENT.HSF.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). CCSS.MATH.CONTENT.HSF.BF.A.1 Write a function that describes a relationship between two quantities.* CCSS.MATH.CONTENT.HSF.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. CCSS.MATH.CONTENT.HSF.BF.A.1.C (+) Compose functions. CCSS.MATH.CONTENT.HSF.BF.B.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. CCSS.MATH.CONTENT.HSG.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</p>		<ol style="list-style-type: none"> <li>11. Using a graphing calculator to determine the maximum or minimum value of a function in an appropriate domain</li> <li>12. Graphing polynomial and rational functions and inequalities with and without technology</li> <li>13. Finding slant, vertical and horizontal asymptotes and holes</li> <li>14. Compute with imaginary numbers</li> <li>15. Determining the zeros of a polynomial function</li> <li>16. Finding the domain of polynomial, rational, radical functions</li> <li>17. Graphing piecewise functions</li> </ol>
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