Unit C - *Chance*

Overview

In this unit, we discuss the basic ideas and methods of probability. Our goal is not just to help students answer questions like "What's the probability that you get no heads if you toss a fair coin 5 times?" We aim to show students the role that probability plays in statistical inference. Contrast the previous question with this one: "Suppose you toss a coin five times and get no heads. Is the coin fair?" That's a statistics question, but you need to understand probability to answer it.

Probability is about much more than coins, dice, and cards. It's about making decisions in the face of uncertainty. People use probability to assess the results of drug tests, to determine the strength of certain kinds of evidence in a court case, to set insurance premiums, to choose an investment strategy, and to weigh the risks and benefits of medical treatment options. Of course, probability also plays an integral role in games of chance, from state and national lotteries to casino favorites like slot machines, craps, roulette, and Texas Hold 'Em. In Unit C, we try to strike a balance between applications involving games of chance (which motivated the study of probability in the first place) and interesting uses of probability in everyday life.

21st Century Capacities: Analyzing

Stage 1 - Desired Results

ESTABLISHED GOALS/ STANDARDS

MP 1 Make sense sense of problems and persevere in solving them
MP2 Reason abstractly and quantitatively
MP5 Use appropriate tools strategically
MP7 Look for and make use of structure
MP8 Look for and express regularity in repeated reasoning

Understand independence and conditional probability and use them to interpret data

CCSS.MATH.CONTENT.HSS.CP.A.1
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the

Transfer:

Students will be able to independently use their learning in new situations to...

1. Represent, interpret, and draw conclusions in numbers, data and objects. (Analyzing)
2. Demonstrate fluency with math facts, computation and concepts.
3. Justify reasoning using clear and appropriate mathematical language.

Meaning:

UNDERSTANDINGS: *Students will understand that:*

1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions or make predictions.
2. Mathematicians apply the mathematics

ESSENTIAL QUESTIONS: *Students will explore & address these recurring questions:*

A. What role does probability play in statistical inference?
B. How can we make decisions in the face of uncertainty?

outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). CCSS.MATH.CONTENT.HSS.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. CCSS.MATH.CONTENT.HSS.CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and*B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. CCSS.MATH.CONTENT.HSS.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* CCSS.MATH.CONTENT.HSS.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **Use the rules of probability to compute probabilities of compound events.** CCSS.MATH.CONTENT.HSS.CP.B.6	they know to solve problems occurring in everyday life. 3. Mathematicians create or use models to examine, describe, solve and/or make predictions.	
	Acquisition:	
	Students will know… 1. The basic rules of probability (p331) 2. The general addition rule for two events 3. The notation for complement, intersection and union of events 4. The notation for conditional probability 5. Two mutually exclusive events can never be independent 6. The general multiplication rule 7. How to find a conditional probability 8. The multiplication rule for independent events 9. The law of large numbers 10. Properties of the Normal distribution 11. Multiplication counting principle 12. The four conditions for a binomial setting (Binary? Independent? Numbers? Success?) 13. Vocabulary: probability, random, independent, probability model, sample space, event, complement, intersections, union, mutually exclusive, disjoint, conditional probability, independent, random variable, probability distribution, sampling distribution, factorial, binary, binomial distribution	*Students will be skilled at…* 14. Using random digits from a table, calculator, or computer software to imitate chance behavior 15. Designing and using a simulation 16. Reading, finding probabilities from, and creating a two way table, Venn diagram, tree diagram 17. Reading and creating a probability distribution 18. Finding the expected value of a random variable 19. Finding expected values by simulation 20. Finding permutations and combinations 21. Using the Binomial theorem

Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **Calculate expected values and use them to solve problems** CCSS.MATH.CONTENT.HSS.MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. CCSS.MATH.CONTENT.HSS.MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. CCSS.MATH.CONTENT.HSS.MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* CCSS.MATH.CONTENT.HSS.MD.A.4 (+) Develop a probability distribution for a random variable		

defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **Use probability to evaluate outcomes of decisions** CCSS.MATH.CONTENT.HSS.MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. CCSS.MATH.CONTENT.HSS.MD.B.5.A Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* CCSS.MATH.CONTENT.HSS.MD.B.5.B Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* CCSS.MATH.CONTENT.HSS.MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		